Quantum Feedback Control Outside of the Controlled System

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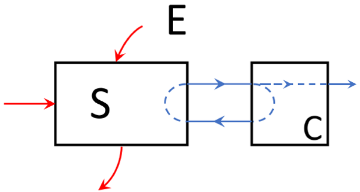
Contents

Classical feedback control

Control theory is concerned with methods to manipulate the evolution of dynamical systems. Feedback is its essential tool, as it allows to monitor the system and adjust its parameters based on the estimates of its current state. The essential parts of the control problems are:

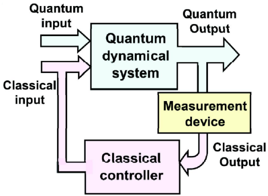
- The system of interest itself (S)
- The controller (C)
- The inputs and outputs from and to environment (E), representing various uncontrolled interaction channels



Quantum feedback control – the two paradigms

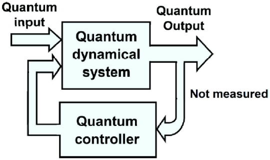
Measurement-based feedback

The feedback action is based on the outcomes of discrete or continuous measurement of the system.



Coherent feedback

The system of interest is made to interact with ancilla quantum system(s), specially engineered to drive the system into target state.



Full quantum loop

Applications

- Quantum state engineering
- Fighting decoherence
- Quantum parameter estimation
- Quantum state discrimination

Unravelling of a quantum operation

Quantum operations

Quantum operation, also known as completely positive trace-preserving map (CPTP) is the most general type of mapping between quantum states: $\hat{\varrho} \to \Lambda\hat{\varrho} = \sum_i \hat{L}_i \hat{\varrho} \hat{L}_i^\dagger$; $\sum_i \hat{L}_i^\dagger \hat{L}_i = \hat{1}$. $\{\hat{L}_i\}$ are known as Kraus operators. The map Λ may be interpreted as generalized measurement, with index i representing a certain measurement outcome.

Markovian master equation

When considering time evolution of a quantum state, it is possible to find a generator of CPTP maps in the following form (the Lindblad equation): $\hat{\varrho}(t+t_0) = e^{\mathcal{L}t}\hat{\varrho}(t_0) \Rightarrow \frac{d}{dt}\hat{\varrho} = \mathcal{L}\hat{\varrho} = -\imath[\hat{H}, \hat{\varrho}] + \sum_i \left(\hat{A}_i \hat{\varrho} \hat{A}_i^\dagger - \frac{1}{2}\hat{A}_i^\dagger \hat{A}_i \hat{\varrho} - \frac{1}{2}\hat{\varrho}\hat{A}_i^\dagger \hat{A}_i\right)$

Unravelling

The set of Kraus operators uniquely defines the quantum operation, but not vice versa. A unitary transform U applied to $\{\hat{L}_i\}$ yields another set $\{\hat{L}_i(U) = U_{ij}\hat{L}_j\}$ which defines exactly the same operation. Fixing a specific choice of U is known as unravelling of a quantum operation (Carmichael, 1993).

Hybrid quantum-classical systems

Definition

A hybrid system is a complex of coupled quantum and classical systems (Diosi, 2014; Blanchard & Jadczyk, 1997) The state of its classical part is described by a classical random variable σ, while the state of a quantum part is represented by a set of positive operators $\hat{\varrho}^{(\sigma)}$ (hybrid densities), so that $\sum_\sigma \hat{\varrho}^{(\sigma)} = \hat{\varrho}$ yields its total state and $p^{(\sigma)} = Tr(\hat{\varrho}^{(\sigma)})$ is the probability of the classical system to be in a σ-state.

Master equations

The quantum system may be coupled to additional reservoir. However, its evolution also affects the classical subsystem, so the Hamiltonian as well as the Lindblad operators act on a σ-space:

$\hat{H} = \sum_\sigma \hat{H}^{(\sigma)} \otimes |\sigma\rangle\langle\sigma|$; $\hat{A}_i = \sum_{\sigma,\sigma'} \hat{L}_i^{(\sigma)} \otimes |\sigma\rangle\langle\sigma'|$

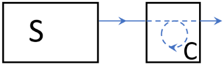
$$\frac{d}{dt}\hat{\rho} = -\imath[\hat{H},\hat{\rho}] + \sum_i \left(\hat{A}_i\hat{\rho}\hat{A}_i^\dagger - \frac{1}{2}\hat{A}_i^\dagger\hat{A}_i\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{A}_i^\dagger\hat{A}_i \right)$$

Tracing over the classical variables yields

$$\frac{d}{dt}\hat{\rho}^{(\sigma)} = -\imath[\hat{H}^{(\sigma)},\hat{\rho}^{(\sigma)}] + \sum_{i,\sigma'} \left(\hat{L}_i^{(\sigma)}\hat{\rho}^{(\sigma')}\hat{L}_i^{(\sigma)\dagger} - \frac{1}{2}\hat{L}_i^{(\sigma')\dagger}\hat{L}_i^{(\sigma')}\hat{\rho}^{(\sigma)} - \frac{1}{2}\hat{\rho}^{(\sigma)}\hat{L}_i^{(\sigma')\dagger}\hat{L}_i^{(\sigma')} \right)$$

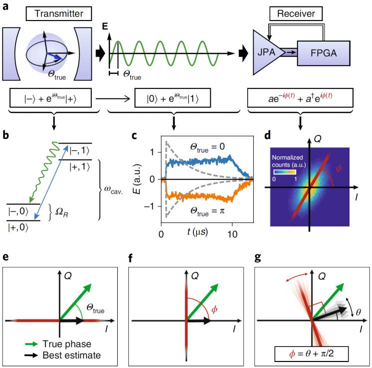
Idea of unravelling-based feedback

Unlike classical systems, it is possible to consider almost total control of quantum system's interaction with environment. This can be done by a (classical) controller, thus making system a hybrid one. One can also organize a feedback loop that would control the unravelling of quantum system's interaction with environment.



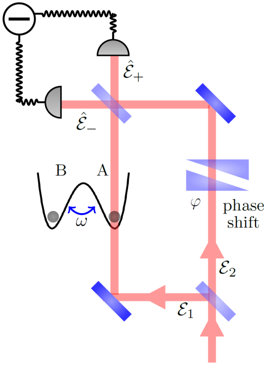
The feedback loop is closed outside the system, and yet is capable of modifying the system's evolution. This idea utilizes quantum properties of the system and cannot be implemented in classics.

A similar idea in the context of adaptive phase measurement:



Martin *et al.*, Nat. Phys. **16**, 1046 (2020).

Interferometric probing of two-mode atomic BEC



Tomilin & Il'ichov, Ann. Phys. 528, 619 (2016)

BEC model

We consider two symmetric localized BEC modes (A and B) of non-interacting atoms. The most essential part of the model is the non-zero tunneling rate ω between the modes, and it is the only term that remains in the interaction picture:

$$\hat{H} = \omega\left(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}\right).$$

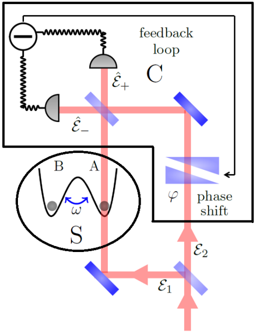
Decoherence model

One of the BEC modes is probed by a non-resonant light field. It interacts with the atoms in a dispersive regime, gaining a phase shift proportional to the number of atoms in mode A:

$$\frac{d}{dt}\hat{\varrho} = -\imath[\hat{H}, \hat{\varrho}] + \sum_{\sigma=\pm}\left(2\hat{\mathcal{E}}_\sigma(\varphi)\hat{\varrho}\hat{\mathcal{E}}_\sigma^\dagger(\varphi) - \hat{\mathcal{E}}_\sigma^\dagger(\varphi)\hat{\mathcal{E}}_\sigma(\varphi)\hat{\varrho} - \hat{\varrho}\hat{\mathcal{E}}_\sigma^\dagger(\varphi)\hat{\mathcal{E}}_\sigma(\varphi)\right)$$

$$\hat{\mathcal{E}}_\pm(\varphi) = \frac{1}{\sqrt{2}}\left(e^{\imath\chi\hat{a}^\dagger\hat{a}} \pm e^{\imath\varphi}\right)$$

Introducing unravelling-based control



BEC constitutes a quantum part (S) of a hybrid system, while controlled phase shift φ along with the feedback actuator constitutes its classical part (C).

Feedback

The feedback action is initiated by each successful photodetection. It sets the value of phase shift φ to one of the two pre-defined values $\varphi_\pm$, depending on the type of registered photodetection, effectively modifying unravelling:

$$\begin{pmatrix} \hat{\mathcal{E}}_+(\varphi_{\sigma_1}) \\ \hat{\mathcal{E}}_-(\varphi_{\sigma_2}) \end{pmatrix} = U(\varphi_{\sigma_1}, \varphi_{\sigma_2}) \cdot \begin{pmatrix} \hat{\mathcal{E}}_+(0) \\ \hat{\mathcal{E}}_-(0) \end{pmatrix};$$

Master equations

Introducing hybrid densities for BEC $\hat{\varrho}^{(\pm)}$, the following set of coupled master equations can be obtained:

$$\frac{d}{dt}\hat{\varrho}^{(+)} + \imath[\hat{H}, \hat{\varrho}^{(+)}] =$$
$$\sum_{\sigma=\pm}\left(2\hat{\mathcal{E}}_+(\varphi_\sigma)\hat{\varrho}^{(\sigma)}\hat{\mathcal{E}}_+^\dagger(\varphi_\sigma) - \hat{\mathcal{E}}_\sigma^\dagger(\varphi_+)\hat{\mathcal{E}}_\sigma(\varphi_+)\hat{\varrho}^{(+)} - \hat{\varrho}^{(+)}\hat{\mathcal{E}}_\sigma^\dagger(\varphi_+)\hat{\mathcal{E}}_\sigma(\varphi_+)\right),$$
$$\frac{d}{dt}\hat{\varrho}^{(-)} + \imath[\hat{H}, \hat{\varrho}^{(-)}] =$$
$$\sum_{\sigma=\pm}\left(2\hat{\mathcal{E}}_-(\varphi_\sigma)\hat{\varrho}^{(\sigma)}\hat{\mathcal{E}}_-^\dagger(\varphi_\sigma) - \hat{\mathcal{E}}_\sigma^\dagger(\varphi_-)\hat{\mathcal{E}}_\sigma(\varphi_-)\hat{\varrho}^{(-)} - \hat{\varrho}^{(-)}\hat{\mathcal{E}}_\sigma^\dagger(\varphi_-)\hat{\mathcal{E}}_\sigma(\varphi_-)\right).$$

Analyzing steady-state distributions

Rapid decoherence

Steady-state solutions of the master equations may be obtained in the rapid decoherence limit $|\omega| \ll 1$. The natural basis for solution would be $|n\rangle \doteq |n\rangle_a \otimes |N-n\rangle_b$: $\hat{\varrho} = \sum_{n=0}^{N} \Big(p_n |n\rangle\langle n| + q_n |n+1\rangle\langle n| + \overline{q}_n |n\rangle\langle n+1| \Big)$ (the same relations hold for $\hat{\varrho}^{(\pm)}$).

Kullback information

It would be interesting to compare steady-state distributions with and without feedback. To measure their difference quantitatively we will be using Kullback-Leibler divergence (Kullback information):

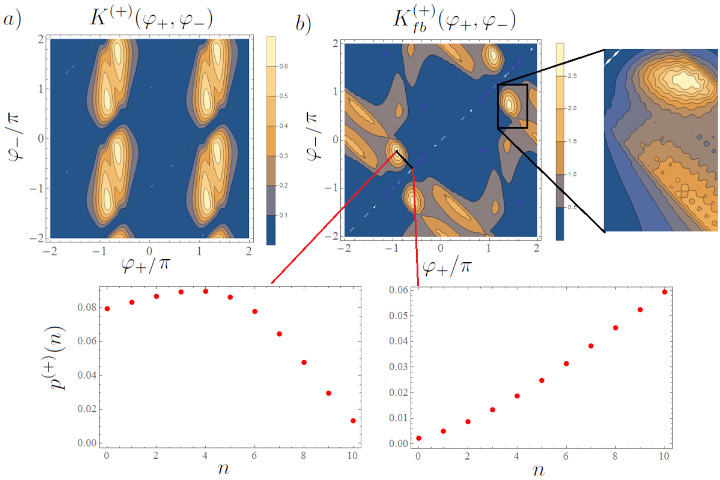
$K \equiv \sum_n p_1(n) \cdot \ln \frac{p_1(n)}{p_2(n)}$.

Specifically, we will be comparing the hybrid distributions $p^{(\pm)}(n)$ with the uniform distribution $p_n = \frac{1}{N+1}$ and the no-feedback distributions:

$$K^{(\pm)}(\varphi_+, \varphi_-) = \sum_{n=0}^{N} P_n^{(\pm)}(\varphi_+, \varphi_-) \cdot \ln \Big(P_n^{(\pm)}(\varphi_+, \varphi_-) \cdot (N+1) \Big),$$

$$K_{fb}^{(\pm)}(\varphi_+, \varphi_-) = \sum_{n=0}^{N} P_n^{(\pm)}(\varphi_+, \varphi_-) \cdot \ln \left(\frac{P_n^{(\pm)}(\varphi_+, \varphi_-)}{P_n^{(\pm)}(\frac{\varphi_+ + \varphi_-}{2}, \frac{\varphi_+ + \varphi_-}{2})} \right).$$

Results



$\chi = \pi/20$, $\omega = 0.07$, $N = 10$

Conclusion and outlook

Results

- Suggested a new scheme of quantum feedback control
- Demonstrated its ability to control a steady-state of a model two-mode BEC system
  Results published in JETP Lett. **116**, 625 (2022)

Directions for further research

- Test the same feedback protocol in different setups
- Try more elaborate versions of the scheme (e.g. based on a total or truncated history of events).
  $\hat{\varrho}^{(\sigma)} \rightarrow \hat{\varrho}^{(\sigma_1 \dots \sigma_n)}$